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NEW YORK UNIVERSITY

WASHINGTON SQUARE COLLEGE

MATHEMATICS DEPARTMENT RESEARCH GROUP

# FINAL REPORT

JUNE, 1948

Contract No. W 28-099-ac-170
with
Watson Laboratories, Air Materiel Command
United States Air Force

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Subject:

The mathematics of wave guides and cavities in the range of 11,000 to 33,000 megacycles.

Submitted by H. R. Cooley

H. R. Cooley
Technical Editor

Morris Kline

Morris Kline
Project Director

CONTENTS

## I. Introduction

The group whose collective work is summarized in this Report began to be organized in July of 1946 to investigate basic mathematical problems in the field of high frequency electromagnetic phenomena associated with wave-guides and cavities. The composition of the group was not constant during the two year period of the contract, some members being added to it at various times as others left for academic positions elsewhere.

It was never the expectation of the group that anyone would easily improve upon a field of analysis which has been profoundly studied, in one or another form, during the past century. Rather, it was hoped that the group would be able by its experience in mathematics to contribute to a better understanding of the mathematical background and techniques available for current engineering problems, and to stimulate a wider group of workers to an interest in these problems.

The principal mathematical success of the group, and ultimately the principal object of attention, was the study of a variety of geometrical models of the helical wave-guide and the various-order modes of associated electromagnetic fields. This work will be reported in the next section. Later sections will treat some of the other problems, listed in the Table of Contents, on which appreciable time of group members was spent.

## II. The Helical Wave-Guide.

In two Reports (2,3) (a list of the Research Reports by members of the group is contained in the References on page (15 ) on the helical guide, the basic model is that of a circular cylindrical surface at each point of which there is defined a fixed, constant helical direction along which the surface is perfectly conducting, but such that at right angles to this direction the surface is non-conducting. The model achieves the character of current flow to be expected on a helical wire, and has appealed to a number of investigators. It appears in the literature at least as early as 1926[(11)]. The proper boundary conditions at the wall of the guide require the vanishing of the tangential electric field in the helical direction, and the continuity of the corresponding component of magnetic vector.

It seemed desirable to postulate the vanishing of this latter component, and this was done in the first of the Reports submitted. This condition isolates the fields within the guide and outside it, so that they may be treated independently, and leads to a very material simplification of the mathematical analysis. In a subsequent Report this restriction was abandoned. Some discussion of these Reports is given in the succeeding sections (#1,#2).

Since the mathematical advantage to be drawn from the more restrictive hypothesis is great, and the gain in realism by its omission is not as clear cut as might at first be supposed, it was decided to investigate the naturalness of the alternative boundary conditions, from the point of view of non-isotropic media.

The model chosen for this project is a generalization of the Brillouin helical guide, and consists in a pair of parallel infinite plates of infinite thickness of a non-isotropic material. Between these "half-space" plates is a dielectric. Within the plates there is a preferred "horizontal" direction in which they are infinitely conducting, normal to which they are non-conducting. It is found that when the electromagnetic parameters in the bounding material are chosen to agree with those of the dielectric in the "vertical" direction (i.e. perpendicular to the dielectric), then the component of field in the preferred("helical") direction is governed by the wave-equation. It is interesting that this is not the case for a more general choice of the parameters.

Now, if one allows the conductivity (in the preferred direction) to become infinite, one is led to the boundary condition that the "helical" component of magnetic field is continuous but not vanishing at the walls of the guide. This constitutes, in some measure, a justification of the boundary conditions of Research Report No. 170-3.

This project was subsequently extended, by the choice of other models closely related to the one described, in order to study the effects on the fields of finite conductivity in the wall, and of wall-thickness. The study is nearing completion, and will be submitted presently[8].

A very hopeful approach, now well under way, to a study of the fields associated with a helical wire begins with the model of a zero thickness wire coiled in the shape of a helix. The wire is assumed infinitely conducting, and it is supposed that there has been set up along it a progressing wave of current.

It is clear that the helix will be a singular locus for the electromagnetic fields, with consequent difficulties for the mathematical analysis. Most of these seem, now, to have been overcome.

The study centers around the following function:

$$\varphi(\beta, \rho) = \sum_{n=-\infty}^{\infty} K_n\left(\sqrt{(\beta + \frac{n}{a})^2 - k^2}\ \rho\right) I_n\left(\sqrt{(\beta + \frac{n}{a})^2 - k^2}\ a\right)$$

The series is convergent for $\rho$ different from a (the radius of the helix) but diverges as $\rho$ approaches a. One finds that the phase velocity along the guide is given by the equation:

$$\frac{\alpha^2(\beta^2 - k^2)}{a^2 k^2} = \lim_{\rho \to a} \frac{\varphi(\beta + \frac{1}{\alpha}, \rho) + \varphi(\beta - \frac{1}{\alpha}, \rho)}{\varphi(\beta, \rho)}$$

It is believed, but the proof is quite intricate and requires careful scrutiny, that the limit on the right is equal to 1. This gives the phase velocity

$$\beta = \frac{k\sqrt{a^2 + \alpha^2}}{\alpha} \qquad ,$$

which is what one should have if the wave travels along the wire at the speed of light in free space.

The work has progressed to a point where it is sufficiently interesting to merit attention, and at whatever stage of development it may be, within the next month, it will be submitted as a Research Report[9].

The task was undertaken of formulating the mathematics of the exact helical guide. Here the model is an actual helical rod of infinite conductivity. It is assumed, moreover, that in some manner a wave of current has been set up on this helical surface whose direction at every point is perpendicular to the circular cross-sections of the surface.

For this model it is something of a tour de force merely to set up the field expressions and obtain the discriminantal equation from which the phase velocity is to be determined. This has been done, but it is not expected that

the extremely complicated expressions will yield results without very much more labor.

1. We shall give here a brief summary of Special Report No. 170-2, rereferred to in the introductory paragraphs of this section of the present Report. The methods of that paper, appropriately extended, form the basis for the treatment of the more difficult problem discussed below in #2.

The z-components of electromagnetic field on the inside of the circuoar cylindrical "helical" guide are represented in the form:

$$E_z = \sum_{n=-\infty}^{\infty} J_n(\gamma r) A_n F_n \quad ,$$

$$H_z = \sum_{n=-\infty}^{\infty} J_n(\gamma r) B_n F_n \quad ,$$

with $F_n = \exp(in\theta + i\beta z - i\omega t)$

$$\gamma^2 = k^2 - \beta^2 \quad ,$$

$k = 2\pi/\lambda_o$ , where $\lambda_o$ is the free-space wave length,

and $A_n$ and $B_n$ are numerical coefficients.

The other components are determined from those above in the usual way. At each point $(a, \theta, z)$ of the guide wall $r = a$, the fixed helical direction of infinite conductivity is denoted by the vector

$$\underline{s} = a\underline{v}_\theta + \alpha \underline{v}_z \quad ;$$

here $\underline{v}_\theta$ and $\underline{v}_z$ are orthogonal unit vectors tangent to the guide, $\alpha = d/2\pi$, and d is the distance between turns.

The boundary conditions assumed in the Report are

$$\underline{E} \cdot \underline{s} = 0 \quad \text{and} \quad \underline{H} \cdot \underline{s} = 0.$$

These conditions are identically fulfilled in $\theta$, z, and t. Therefore, for each $n = 0, \pm 1, \pm 2, \ldots$, one obtains a pair of homogeneous equations in $A_n$ and $B_n$

and then as consistency condition the equation:

$$(\alpha - n\beta/\gamma)^2 J_n^{\,2}(\gamma a) = (ka/\gamma)^2 J_n'^{\,2}(\gamma a) \quad .$$

This can be expressed in the more convenient form:

$$\frac{\alpha}{a}\,\frac{1}{ak} = \frac{n\beta}{u^2 k} \pm \frac{J'_n(u)}{uJ_n(u)} \quad , \text{ with } u = \gamma a \quad . \tag{1.1}$$

For each integer n, and to each solution u of 1.1, there corresponds a natural mode of the guide with $A_n = \pm i\eta$ $B_n = 1$. The choice of sign in $A_n$ corresponds to the choice in 1.1; $\eta = \sqrt{\mu/\varepsilon}$ , the resistance of the dielectric medium. No transverse magnetic or transverse electric modes can exist in this guide, and the modes associated with positive n differ from those for the corresponding negative integer, in distinction to the situation in the usual cylindrical guides.

For non-attenuated modes $\beta$ must be real, and hence $\gamma$ must be pure imaginary (I-modes) <u>or</u> real valued and less than k (R-modes). For R-modes the phase velocity along the axis is greater than the velocity of light in free space, for I-modes it is less. There are only a finite number of R- and I-modes for each integer n. It is very plausible that the totality of modes discovered in this paper is a complete set, in the sense that any field existing in the guide is a linear combination of them. This can be demonstrated in the case n = 0. It follows, for this case at least, that the guide has the remarkable property that it admits certain <u>attenuated</u> waves ( $\beta$ complex) which do not dissipate energy along the guide.

It is the I-mode, and in particular the zero-order mode, which is of principal current interest in the application of the helical guide as amplifier of an electron beam current. This mode is discussed in considerable detail. Such a mode exists in the guide if and only if

$$\frac{\alpha}{a}\,\frac{1}{ak} < \frac{1}{2} \ , \tag{1.2}$$

and when this is the case only one zero-order mode is possible. When (1.2) is fulfilled, and moreover <u>ak</u> is small compared to one, the phase velocity of the zero-order mode is isolated from that of all other I-modes, and the R-modes are automatically eliminated.

When the quantity on the left of 1.2) is less than 0.2, it is shown that to first approximation the wave-length along the tube is

$$\lambda_0 \alpha / (a^2 + \alpha^2)^{1/2} ,$$

which is precisely what one would get if the wave followed the helical winding with its free space velocity.

In general, there are a finite number of R-modes possible for each of a finite number of integers n. In addition, there exist either one or two I-modes for each $n < 0$, and none, one, or two I-modes for each $n \geq 0$, depending on the choice of design parameters k, d , and a.

It is part of the object of the paper to study the problem of optimum design of the guide so that the modes which can exist in it are effectively separated with respect to phase velocity. Accordingly, the right side of 1.1) is investigated as a function of the integer n, and for each n as a function of u. Preliminary theorems of some mathematical interest are devoted to an analysis of the function

$$X_n = \frac{J'_n(u)}{u\,J_n(u)} , \quad u = \gamma a, \tag{1.3}$$

through the differential equation of Ricatti type:

$$\frac{dX_n}{du} = - uX_n^2 - \frac{2}{u} X_n - \frac{1}{u} + \frac{n^2}{u^3} .$$

A check of the theory of this idealized helix against some available experimental data gave the following results. For a free space wave length of 82 cm., a spacing between turns of 2 cm. and a helical radius of 3 cm. (inner) and 4.3 cm. (outer), it was necessary to assume in the theory for the idealized helix a cylinder radius of 4.5 cm. in order to obtain the experimentally determined phase velocity of the actual single wire helix. In another instance, for a free space wave length of 10 cm., a spacing between turns of 0.25 cm., and a helical

radius of 2.6 mm. (inner) and 3.8 mm. (outer) it was necessary to assume a cylinder radius of 4 mm. in order to achieve agreement.

2. In a subsequent Research Report(3,7) which was described earlier and will be briefly summarized here, the more general boundary conditions imposed upon the cylindrical "helical" guide lead to the following discriminantal equation for the determination of phase velocities:

$$\frac{\delta}{\alpha} = -\frac{n}{v^2}\left[1 + \left(\frac{v}{\alpha}\right)^2\right]^{1/2} \pm \left[-\frac{I_n'(v)\,K_n'(v)}{v^2 I_n(v)\,K_n(v)}\right]^{1/2} . \quad (2.1)$$

Certain changes in notation appear in this Report; here $\alpha = \frac{2\pi a}{\lambda_0}$ and $\delta = \frac{d}{2\lambda a}$ , so that the left sides of 1.1) and 2.1) are the same physical quantity. Comparison of the right sides shows that the more general boundary condition has immeasurably complicated the analysis.

The term on the right of (2.1) is the geometric mean of the functions:

$$X_n = \frac{I_n'(v)}{vI_n(v)} , \text{ and } y_n = -\frac{K_n'(v)}{vK_n(v)} . \quad (2.2)$$

The first of these functions is important for the fields interior to, the second for the exterior fields to the helical guide treated in the preceding Report. Much of the present Report is, of necessity, concerned with these functions.

The equation 2.1) has no imaginary solutions. Each real solution corresponds to a mode with phase velocity along the cylinder smaller than the free-space wave velocity. The "real" modes corresponding to $n = 0$ and $|n| \geq 3$ are rigorously analyzed. The cases $n = \pm 1, \pm 2$, form a more intractable set and graphical evidence is presented for the results. These results are considerably reinforced by a recent Addendum(7) to this Report.

The theory of this Research Report checked against the experimental data already referred to gives agreement with observed phase velocities for a choice of cylinder radius lying between the inner and outer radii of the single wire helix: 4.1 cm. for the first experiment cited and 3.1 cm. for the second.

## III. Coordinate Systems and Guide-Shapes.

A goal which was not achieved was that of making effective use of the twilight class of orthogonal coordinate systems in which the Laplace euqation can be solved by separation of variables and the wave equation cannot. The incentive was threefold. First, a wide variety of striking and useful shapes occur among the coordinate surfaces of these coordinate systems. Second, these shapes are always contained in a one-parameter family of surfaces through which a profound modification of shape transpires. This will be clear to anyone who has a glance at some of the illustrations in Bocher's "Reihenentwickelungen ...."[10]. Third, it did not seem unreasonable that if the wave equation could be solved in coordinate-systems in which the technique of separation of variables was not applicable, such coordinate systems would be found among these in which the Laplace equation was separable. A simple example of this type of coordinate system, and one which might well serve as a test case, is obtained by an inversion in the plane (transformation by reciprocal radii) of the elliptical orthogonal coordinates. The third space-coordinate is supplied by the z-axis.

However, even in this apparently simple case the wave equation proved intractable and no help was found in the integral equations to which one was led. The fact that a Green's function could be found did not advance the solution because of the complexity of this function.

It is the opinion of the group that this field of problems deserves continued study. It should be kept in mind that it is possible that the problem may finally resolve itself into an extensive project for numerical calculation.

## IV. Limiting Configurations.

It is well known that with some of the orthogonal coordinate systems in common use there are associated limiting configurations of great interest; many of these have been well exploited: e.g., the elliptical disc, the thin rod, the elliptical hole in an infinite plate, etc.

It should be remarked that there is another, apparently less familiar type of limiting configuration consisting of such surfaces as, for example, the following: an ellipsoid with an annular elliptical sheet jutting out from it and partially blocking the ellipsoidal cavity. Such problems, which seem worthy of

attention from the point of view of modifications of cavity shapes can be investigated in the standard coordinate systems.

It seemed worthwhile to consider the simpler, analagous problem, in which a variable quasi-rectangle in circular cylindrical coordinates was moved off to infinity, but so modified as to approach in shape a limiting rectangle. This was adapted to the study of a rather special type of buckling of a rectangular wave-guide (4). The series expansion developed for this problem proved rather tedious for the calculation of high order terms. There appears in this study the following curious expansion of a well-known function, in which however the order of the Bessel functions is proportional to one of the arguments:

$$\frac{\pi}{2}\left[J_\nu(kR)\,N_\nu(ka+kR) - J_\nu(ka+kR)\,N_\nu(kR)\right]_{\nu = cR}$$

$$= \frac{a}{R}\,\frac{\sin x}{x} + \frac{1}{2}\,\frac{a^2}{R^2}\left\{\frac{\sin x}{x} - \left(1+\frac{1}{\gamma}\right)\cos x\right\} + \ldots.$$

In this exapnsion about $R = \infty$, $c$ is a constant,

$$x = -k^2a^2\gamma \qquad \text{and} \quad \gamma = \frac{c^2}{k^2} - 1 \qquad .$$

One higher order term was obtained, but this proved too complex to command interest.

A study of another type limiting configuration was completed (1.2), in which there was investigated the transition of a coaxial circular cylindrical guide to a cylindrical guide, as the radius of the inner tube approaches zero. The Report was principally concerned with the dependence upon the inner radius of the family of cut-off frequencies associated with the various-order modes of the coaxial guide (with small inner radius).

## V. Variational Techniques.

Every effort was made to keep abreast of recent developments in variational techniques applicable to Maxwell's equations. This was not always a simple matter, particularly in the first year of this undertaking when work that had been

developed during the war remained largely inaccessible. However, some of that work is now beginning to appear.

One application of such new techniques is made in Research Report No. 170-5. This paper studies the effect upon the principal field of a rectangular guide of the insertion lengthwise into it of a narrow, finite, dielectric block.

## VI. Lossy Walled Rectangular Guide.

No method was found for the solution of the interesting and possibly important problem of analyzing the fields in a rectangular guide with poorly conducting walls. It was felt that for quite lossy walls the standard procedure of calculating power loss on the basis of the field configurations of a perfectly conducting guide might be seriously in error. The difficulty here lies in finding a manageable formulation of the field exterior to such a guide so that the boundary conditions can be set up in workable form. Even the problem of the fields exterior to a perfectly conducting rectangular guide appeared to be extremely difficult.

## VII. Bibliography

In the course of study by member of the group and the exchange of ideas in seminars and at a number of invited addressed by specialists in some field of electromagnetism, there began to be collected a substantial bibliography. This was subsequently augmented by a systematic search through periodicals and references, and is submitted to the Laboratories as Special Report No. 170-6.

## VIII. Education

Serious attention was given to one of the objectives of government subsidization of university research, namely, the development of the scientific potential of the country. Considerable training in the field of wave-guide and cavity theory was afforded a few younger, pre-doctorate mathematicians, and much interest in the application of mathematics to electromagnetism stimulated among the students and faculty at New York University.

Two offerings in the Graduate School also aided in stimulating interest in the applications of mathematics and electromagnetic theory to wave guide and

cavity theory. During the academic year 1946-47 a seminar in electromagnetic problems and during the year 1947-48 an advanced course in electromagnetic theory were offered. Both were well attended. While these courses were not subsidized by government funds the incentive to offer them and the interest in them on the part of some of the students was motivated by association with the work of this group.

IX. Administrative Report.

1. Correspondence.

a. Accountable Property.

On 13 April we received a letter from Mr. M.L. Wasser, Research Accountable Property Officer, Watson Laboratories, requesting a list of expendable and non-expendable material and the designation of an authorized representative to take responsibility for such material; also Memo No.35-6520-1. On 19 April we submitted to Mr. M.L. Wasser a list of non-expendable material and specimen signature of Morris Kline, as authorized representative, with copies to Vice Chancellor L. E. Kimball, New York University.

We also wrote to Mr. Herbert S. Bennett, Watson Laboratories, requesting information as to the possibility of considering books and journals expendable, also as to the disposition of furniture upon closing of the contracts. On 26 April Mr. Herbert S. Bennett replied, advising us to make a formal proposal to the Contracting Officer, Watson Laboratories, concerning the expendability of books, journals and furniture.

On 12 May Mr. M. L. Wasser returned our lists of non-expendable material, submitted on 19 April, with a request that "miscellaneous books" be itemized in a certified list. As a result of correspondence dated 12,13,18,20 and 21 May, the lists of non-expendable material, including books, was sent to Mr. Wasser 26 May.

Regarding disposition of the furniture, we proposed to Lt. H.W.Liljedahl on 3 June that such material charged to our Contract No. W 28-099-ac-170, terminating 30 June, be transferred to our Contract No. W 28-099-ac-172, which is being extended to 30 September and is expected to be renewed for another two-year period.

b. Borrowed Reports

On 12 May we received a letter from Mr. Herbert S. Bennett concerning the return of three reports which presumably had been loaned to us; also on 13 May a letter was received from Captain Thomas F. Conway on the same subject. Replies to these letters were dated 17 and 18 May, confirming the borrowing of two of these Reports and promising their return. These were returned to Miss Charlotte Sloane, Watson Laboratories.

c. Exchange of Reports and Journals.

On 17 March we wrote to Mr. John Hewitt of the Research Laboratory of Electronics, Massachusetts Institute of Technology, requesting that the Quarterly Progress Reports of the Laboratory continue to be sent us; we have been in regular receipt of these reports.

On 1 April we requested of Professor A. L. Samuel, University of Illinois, details of his recent work on the travelling tube. On 13 May at the request of Prof. A. L. Samuel, we forwarded him a copy of our Research Report No. 170-3; authorization for this action had been received from Lt. Liljedahl, Watson Laboratories, on 11 May.

On 5 May we received from Mr. A. Lederman, Panel on Electron Tubes, New York City, a report on "A Contribution to an Elementary Theory of a Small Signal Amplification by Travelling Wave Tubes" by Ludwig Mayer; this was acknowledged.

Two copies of each of six of our Research Reports were requested by the Naval Research Laboratory on 17 May, and were forwarded on 20 May, following authorization on 24 May from Lt. Liljedahl.

We have received Progress Reports of the Computation Laboratory of the National Applied Mathematics Laboratories for the months of March, April and May.

d. Miscellaneous.

On 1 April we wrote to Mr. Philip Parzen, Federal Telecommunication Laboratories, Inc., requesting permission to visit the Laboratories and discuss problems of mutual interest,(see conferences, below).

On 11 May we received a letter from Lt. H. W. Liljedahl, Watson Laboratories, disallowing claim dated 29 March, 1948 presented in Form 1934 Voucher.

e, Continued Sponsorship of Work.

A change in Directors no longer makes it possible for Watson Laboratories to sponsor the type of research undertaken under Contract No. W 28-099-ac-170. It was the opinion of some engineers of the Laboratories and some members of this group that one phase of this work especially, namely the work on the travelling wave tube, had made good progress and should be continued. A proposal to this effect was favorably endorsed by engineers of the Watson Laboratories and submitted to the Cambridge Field Station of the U. S. Air Force in January.

On 4 June Dr. L. M. Hollingsworth of the Cambridge Field Station offered to underwrite the work for at least another year. At the moment the decision to continue the work is being held in abeyance pending determination of available manpower. Our personnel problem has been somewhat complicated in this connection by an interest shown in the helix as an antenna by engineers of the Naval Research Laboratory who have proposed work on this subject.

2. Conferences:

On 31 March, 1948 Mr. A. Lederman and Mr. S. J. Tetenbaum, of the Panel on Electron Tubes, met here with Professors B. Friedman and H. Malin and Mr. W. Sollfrey of this group and discussed the theoretical problems connected with various types of travelling wave tubes. Emphasis of the discussion was laid on isolating the factors which make possible broadbandedness, high gain, and low noise-to-signal ratio.

On 6 April 1948 Professor B. Friedman and Mr. W. Sollfrey visited the Federal Telecommunications Laboratories, Nutley, N.J. to meet Mr. G. Dewey. Information was gained concerning the research on travelling wave tubes at the Laboratories, especially pertaining to typical tube dimensions. Mr. Dewey described certain problems which he would be interested in seeing investigated.

On 29 April to 1 May, 1948 - Meeting of the American Physical Society in Washington, D.C., was attended by Mr. W. Sollfrey. He was particularly interested in a paper by H. Levine and J. Schwinger on variational solutions of diffraction problems; also a paper by H. Feshbach and J. Schwinger, which discussed a procedure of variation and iteration to obtain the egenvalues of differential equations.

On 13 May Professor B. Friedman and Mr. W. Sollfrey, of this group, and Professor J.J.Stoker of New York University participated in a conference with Mr. H. Harrison, Dr. Marsden and Mr. W. Ament, representing the Office of Naval Research in Washington, D.C. The object of the conference was to ascertain what interest the work of this group might hold for representatives of the Office of Naval Research.

X. Research Reports Nos. 170-1 to 170-9 submitted under Contract No.W28-099-ac-170

*1. Relationship between Coaxial and Cylindrical Wave Guide and Cavity Modes, by Morris Kline.

*1a. Addendum to 170-1, by Morris Kline.

2. A Helical Wave Guide, by Ralph S. Phillips.

3. A Helical Wave Guide II, by Ralph S. Phillips and Henry Malin.

4. Deformation of a Rectangular Guide, by Leo Zippin and Morris Kline.

5. Equivalent Circuits for Narrow Dielectric Blocks in Waveguides, by William Sollfrey.

6. Bibliography, compiled by Thelma Braverman and Jerome Lurye.

7. Investigation of the Exceptional Modes: $n = \pm 1, \pm 2$, by Ralph S. Phillips and Henry Malin. Addendum to No. 170-3.

**8. Propagation in Wave Guides Bounded by Electrically Anisotropic Plates, by Bernard Friedman and Henry Malin.

**9. Helical Thread Wave Guide, by Ralph S. Phillips

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* The substance of this Report has been accepted for publication by the Journal of Mathematics and Physics.

** To be submitted in July.

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References:


10. Bocher, Maxim, Reihenentwickelungen der Potentialtheorie, Teubner, Leipzig, 1894.

11. Ollendorff, Franz, Die Grundlagen der Hochfrequenztechnik, pp. 79-87, Springer, Berlin, 1926.


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